Multiplicity distribution in rapidity gap events

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Jet production in high-energy hadron-hadron collisions is associated with either color exchange or color-singlet exchange processes. In the latter case, a signature of rapidity gap events is expected. We accommodate these two different processes with the clan model. The multiplicity distributions can be well described by simple parametrizations with two free parameters. The differences between color exchange and color-singlet exchange processes lie in their different decay distributions. The randomly produced clans decay with the logarithmic and the geometric distributions for color exchange and color-singlet exchange processes, respectively. The striking excess of low multiplicity events in the color-singlet exchange processes can be well described. [S0556-2821(98)04013-2]

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I. INTRODUCTION

Jet production in high-energy hadron-hadron collisions is typically associated with the color exchange processes, in which a quark or gluon is exchanged between interacting partons. In addition to these color exchange processes, jets can also result from the colorless (color-singlet) exchange processes, in which a photon, W boson, Z boson, or two-gluon colorless is exchanged instead. A signature of two jets separated by a rapidity gap has been proposed for the colorless exchange processes and has been observed in experiment recently [1,2]. The rapidity gap between two jets is taken as a region of rapidity containing no final state particles.

For these two different exchange processes, the resulted multiplicity distributions are similar in shape except at very low multiplicities. Compared to the color exchange processes, the gluon radiation between scattered partons is highly suppressed for the colorless exchange processes [3]. Thus a striking excess of events with low multiplicities is present in the colorless exchange processes.

In this Brief Report, we try to accommodate these two different processes phenomenologically with the clan model. The multiplicity distributions can be well described by simple parametrizations with only two free parameters. The differences between color exchange and colorless exchange processes lie in their different decay distributions. The randomly produced clans decay with the logarithmic and the geometric distributions for color exchange and colorless exchange processes, respectively. The striking excess of low multiplicity events in the colorless exchange processes can be well described.

II. PHENOMENOLOGICAL CLAN MODEL

The multiplicity distribution between jets in the color exchange events is expected to be described by a negative binomial distribution [4]. This parametrization is supported by experimental data and Monte Carlo samples over the full range of multiplicity. In the clan model, the negative bimonial distribution can be simply reproduced by a two-step process [5]. In the first step, the clans are produced randomly and distributed as a Poisson distribution,

$$P_1(n)=\frac{\lambda^n}{n!}e^{-\lambda}. \tag{1}$$

In the second step, each clan decays into final particles independently with a logarithmic distribution

$$P_2(n)\propto\frac{b^n}{n}. \tag{2}$$

A simple constraint is imposed: $P_2(0)=0$, which implies that each clan produces at least one final particle. As the clans are identified by the final particles, a produced clan without decaying into final particles is excluded phenomenologically. The resulted multiplicity distribution is a negative binomial distribution, which is determined by the combinations of $P_1(n)$ and $P_2(n)$, and can be easily calculated through the corresponding generating functions [6]. The two free parameters are λ and b, which can be fitted from the experimental data. Multiplicity distributions from various experimental data can be well described by choosing the appropriate values for λ and b. As their two parameters vary, none of the resulted multiplicity distributions have a significant excess of probabilities at low multiplicities, which also reproduces the feature observed in the color exchange processes.

In contrast, the colorless exchange processes result in a significant excess of probabilities at low multiplicities, especially the void probability which corresponds to the rapidity gap events. We find that the enhancement of low multiplicity events can be effectively accommodated in the clan model by a different choice of the decay distribution $P_2(n)$. With $P_2(n)$ replaced by a geometric distribution

$$P_2(n)\propto b^n, \tag{3}$$

the resulted multiplicity distribution is the Pólya-Aeppli distribution [7], which has also been derived analytically from the Ising model recently [8]. Compared to the negative binomial distributions, the Pólya-Aeppli distributions prescribe an enhancement of probabilities in low multiplicities. Again,

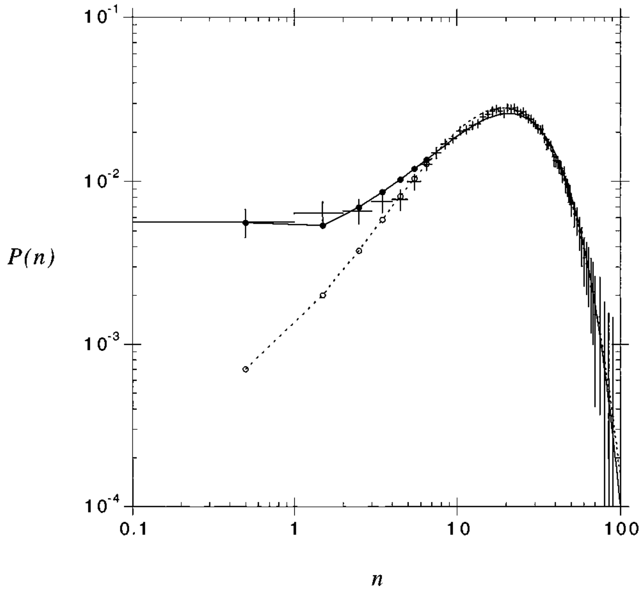

FIG. 1. $P(n)$ versus n (shifted up by 0.5 in multiplicity n) on a log-log scale for multiplicity distribution between the jet edges. Solid and dashed lines are the predictions from the Pólya-Aeppli distribution and the negative binomial distribution, respectively; the full (●) and open (○) symbols denote the values of $P(n)$ for the first few n. Data points are taken from Ref. [2].

there are only two free parameters λ and b, which control the production distribution $P_1(n)$ and decay distribution $P_2(n)$, respectively.

Next, we apply these parametrizations to the experimental data of Fermilab Tevatron for $p\bar{p}$ collisions at $\sqrt{s}=$ 1.8 TeV [2]. As there are only two free parameters, λ and b, we simply fix them by the given values of average multiplicity $\langle n\rangle$ and dispersion $D\equiv\sqrt{\langle n^2\rangle-\langle n\rangle^2}$ of the data. Thus without proceeding the best-fit, the shape of the distribution constitutes a prediction of the models. The results are shown in Fig. 1 for the multiplicity distribution between the cone edges of the two leading jets for the opposite-side data sample. For higher multiplicities, $n>20$, the negative binomial distribution and the Pólya-Aeppli distribution prescribe the same shape. Their difference is in the prediction of low multiplicities. Within the range $10<n<20$, the negative binomial distribution predicts a slightly higher value than the Pólya-Aeppli distribution does. For the low multiplicity events $n<10$, the difference between the prediction of the negative binomial distribution and that of the Pólya-Aeppli distribution can be observed easily. The negative binomial distribution presents no significant excess of probabilities at low probabilities and thus fails to describe the experimental data. In contrast, the Pólya-Aeppli distribution presents a strong enhancement of the probabilities in low multiplicities, especially the void probability. The features of experimental data can be fully reproduced.

For a region well away from the jet edges, we also obtain the similar results. In Fig. 2 we present the multiplicity distribution within a central rapidity interval $|\eta|<1$. Again, the Pólya-Aeppli distribution describes the experimental data much better than the negative binomial distribution does, especially the enhancement of the low multiplicity events. Even without proceeding the best-fit, the description of the data appears to be satisfactory.

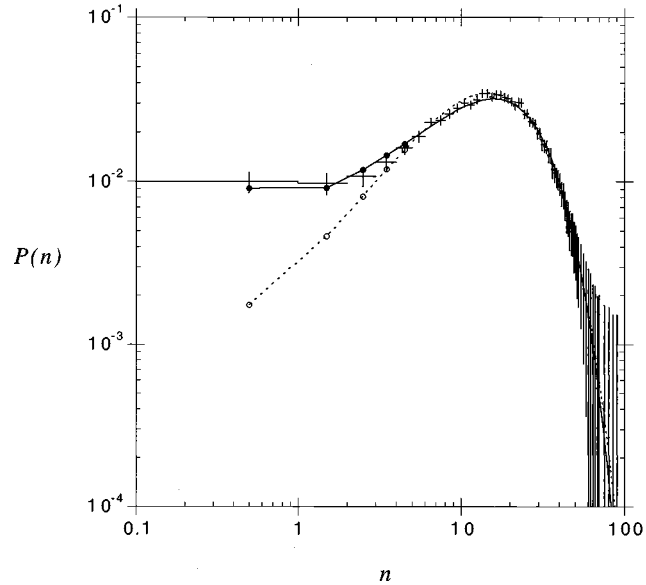

FIG. 2. Multiplicity distribution within the rapidity interval $|\eta|<1$. The symbols are the same as in Fig. 1. Data points are taken from Ref. [2].

III. DISCUSSIONS

Multiplicity distribution of final state particles in the rapidity interval between jets offers a convenient way to distinguish colorless exchange from color exchange processes. The seemingly different distributions can be well accommodated phenomenologically in the clan models with different choices of the decay distributions. With a simple choice of the geometric distribution for colorless events, in contrast to the logarithmic distribution for color exchange processes, the experimental data can be well accounted for.

The main purpose of this paper is to point out that the low-n-enhanced multiplicity distribution in rapidity gap events can be understood in terms of simple assumptions of considerable generality. The occurrence of the Pólya-Aeppli distribution can be found in a general framework of cascading mechanism of particle production. The clan is a group of particles of common ancestry in such a cascade. Assuming the clans to be produced independently (Poisson distribution) and the multiplicity distribution of an average clan is geometric, the overall multiplicity distribution is the Pólya-Aeppli distribution. In contrast, the resulted distribution becomes the negative binomial distribution if the geometric distribution is replaced by the logarithmic one.

In high energy collisions, cascading and fragmentation play an important role in particle production processes. Any attempt to describe the shape of multiplicity distributions must allow for these mechanisms. Both the geometric and the logarithmic distributions occur naturally in the parton shower model where the clans are identified as the bremsstrahlung jets [9]. These distributions correspond to simple

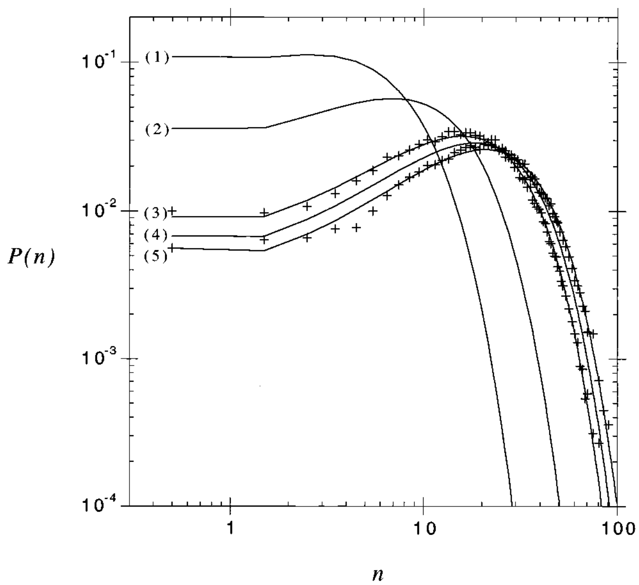

FIG. 3. Multiplicity distributions within different rapidity interval: (1) $\Delta\eta = 0.5$; (2) $\Delta\eta = 1$; (3) $\Delta\eta = 2$; (4) $\Delta\eta = 4$; (5) full range. Data points for curves (3) and (5) are the same as in Figs. 2 and 1, respectively. Errors are not shown.

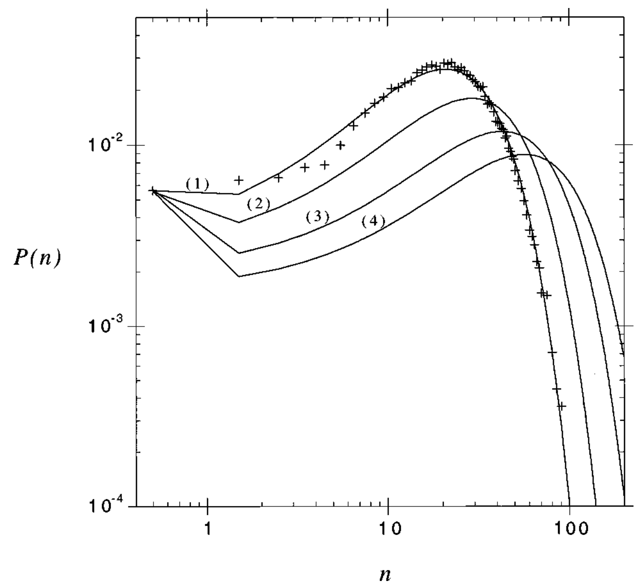

FIG. 4. Multiplicity distributions at higher energies: (1) $\langle n\rangle = 28$; (2) $\langle n\rangle = 40$; (3) $\langle n\rangle = 60$; (4) $\langle n\rangle = 80$. Data points for curve (1) are the same as in Fig. 1. Errors are not shown.

self-similar cascade processes and are expected to be good approximations for QCD jet productions. At first sight, the geometric distribution seems unlikely to be able to prescribe a strongly suppressed radiation as it decreases more slowly than the logarithmic distribution does (at the same value of parameter b). However, after taking into account the proper normalization, the geometric distribution does provide a much smaller probability, i.e., the suppressed radiation, which then results in an enhancement of the probabilities in the low multiplicity region of the rapidity gap events.

As there are only two parameters in the clan models, λ and b, they can be related analytically to the experimental observables $\langle n\rangle$ and $D \equiv \sqrt{\langle n^2\rangle - \langle n\rangle^2}$. For the Pólya-Aeppli distribution, we have

$$\langle n\rangle = \lambda \cdot \frac{1}{1-b}$$

and

$$D^2 = \lambda \cdot \frac{1+b}{(1-b)^2}. \tag{4}$$

For the negative binomial distribution, the relations become

$$\langle n\rangle = \lambda \cdot \frac{b}{1-b} \Big/ \ln\left(\frac{1}{1-b}\right)$$

and

$$D^2 = \lambda \cdot \frac{b}{(1-b)^2} \Big/ \ln\left(\frac{1}{1-b}\right). \tag{5}$$

These equations can be easily inverted to obtain

$$\lambda_{PA} = \frac{2\langle n\rangle^2}{D^2 + \langle n\rangle},$$

$$b_{PA} = \frac{D^2 - \langle n\rangle}{D^2 + \langle n\rangle}, \tag{6}$$

and

$$\lambda_{nb} = \frac{\langle n\rangle^2}{D^2 - \langle n\rangle} \cdot \ln\left(\frac{D^2}{\langle n\rangle}\right),$$

$$b_{nb} = \frac{D^2 - \langle n\rangle}{D^2}, \tag{7}$$

where the subscript "$_{PA}$" and "$_{nb}$" stand for the Pólya-Aeppli and the negative binomial distributions, respectively. To prescribe the same values of $\langle n\rangle$ and D, the parameters of the Pólya-Aeppli distribution are smaller than those of the negative binomial distribution, i.e.,

$$\lambda_{PA} < \lambda_{nb} \quad \text{and} \quad b_{PA} < b_{nb}. \tag{8}$$

Compared to the negative binomial distribution, the clans are less numerous in the Pólya-Aeppli distribution, but their multiplicity is larger [10]. The enhancement of probabilities in the low multiplicity region of the Pólya-Aeppli distribution results from the combined mechanisms of producing a fewer number of clans and each clan decaying into more particles. With more available data, it would be interesting to further check these characteristics leading to the enhancement of probabilities in low multiplicity events.

If the clan structure of the rapidity gaps events follows the same regularity as that of the negative binomial distributions observed in experimental data, the predictions to the distributions in smaller rapidity windows and higher energies are ready to be made. The clan structure of the negative binomial regularity can be summarized as follows. At a fixed energy, both the number of clans and the clan multiplicity increase linearly as the rapidity windows increase from a small value. At wider windows, both numbers saturate to constant values. For a fixed rapidity window, the average number of clans is approximately energy independent over a large energy range. The increase of multiplicity with the increasing of energy is largely due to the corresponding increasing of clan multiplicities. Assuming the same regularity, the predictions of multiplicity distributions at different rapidity windows and at higher energies are shown in Figs. 3 and 4, respectively. It would be interesting to check these predictions with future experimental data.

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